NeMeSiS News 2008

Newsletter of the National Mathematics Summer School

Yow!

Welcome to the 10th edition of NeMeSiS News! We hope that you enjoy reading it. If you have any comments, ideas or submissions for the next issue, you can either tell Terry (terry@maths.usyd.edu.au) or contact me (merryn.horrocks@chilli.net.au). Thank you to everyone who contributed.

Merryn Horrocks (editor)

Beautiful Number Theory

You know that for many years I have said at NMSS that the reason that number theory is the backbone of the course is that it is accessible to an audience from all over the country with necessarily differing experience of mathematics. I always say that number theory is not the most central part of modern mathematics, which is true, nor is it the most active area of research in the subject at the moment, which is also true.

But it is the subject of Archimedes, Fermat, Gauss, Dirichlet, Riemann, Weil, Serre and Deligne. One could add Tao to that list. And many more. In short it has history! I have always tried to cast my lectures in a somewhat historical context, trying to tell students how discoveries were made, and of the scientific and cultural atmosphere in which the subject developed.

Since my retirement four years ago, I have continued teaching at the University of Sydney, and I have just finished a course in Number Theory to senior undergraduates … MATH3062. I have just set the exam and I thought I would tell you about one of the questions which I dreamed up. The three greatest theorems of classical number theory, in my opinion, are:

- Euclid's proof that there are infinitely many primes.
- The Prime Number Theorem PNT, conjectured by Gauss and proved in 1896 by Hadamard and De La Vallée Poussin. It shows that the number of primes less than n is approximately (in a very precise sense) $n / \ln n$.
- The theorem of Dirichlet, which was proved by Dirichlet in 1837. It shows that there are either no primes, exactly one prime or infinitely many primes in any infinite arithmetic progression.

Most of the proofs of PNT and Dirichlet's Theorem are difficult and use the theory of complex variables well beyond the undergraduate mathematics curriculum offered to the students in MATH3062.

In setting the exam, I realised that one question I had constructed in fact proves a partial result on the existence of primes in arithmetic progressions. I can tell you here about it.

First, every prime apart from 2 ends in a 1, 3, 5, 7 or 9. Of course, apart from 2 and 5 every prime ends in a 1, 3, 7 or 9. So there are infinitely many primes whose last digit is a 1, 3, 7, or 9.

I realised that the question I had set for the exam showed that there are infinitely many primes whose *second last* digit is even! It doesn't show that there are infinitely many primes whose *second last* digit is odd unfortunately, though this is true too. In fact, Dirichlet's Theorem shows that both these are true, though it is not fair to Dirichlet to state them as consequences of his beautiful theorem. Still, I'll send a Mars Bar to anyone who can give me an easy proof that there are infinitely many primes whose second last digit is even. I can't think of an easy proof! Maybe I'll set it next January for the NMSS students. They always surprise me, and will do so again!

By the way, here's a secret about my exams. They're always too hard. This one was no exception and to my great regret I had to remove the question to make the exam more doable in the time available. So the MATH3062 students this year won't see that little fact! I am sorry about that, because I really like it! As it happens there is a former NMSS student in that course. This newsletter won't get to her before the exam and there is no help here, now is there?!

Best regards to you all.

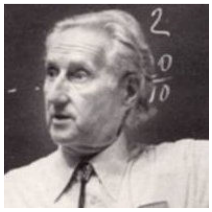
Terry Gagen

NMSS Turns 40!

This year marks 40 years of NMSS (1969-2008). Some 2900 students have attended NMSS as main group students and as EGs since 1978. This edition of NMSS News features some pictures taken over the years. We hope that you enjoy them.

NMSS 1979

Larry Blakers



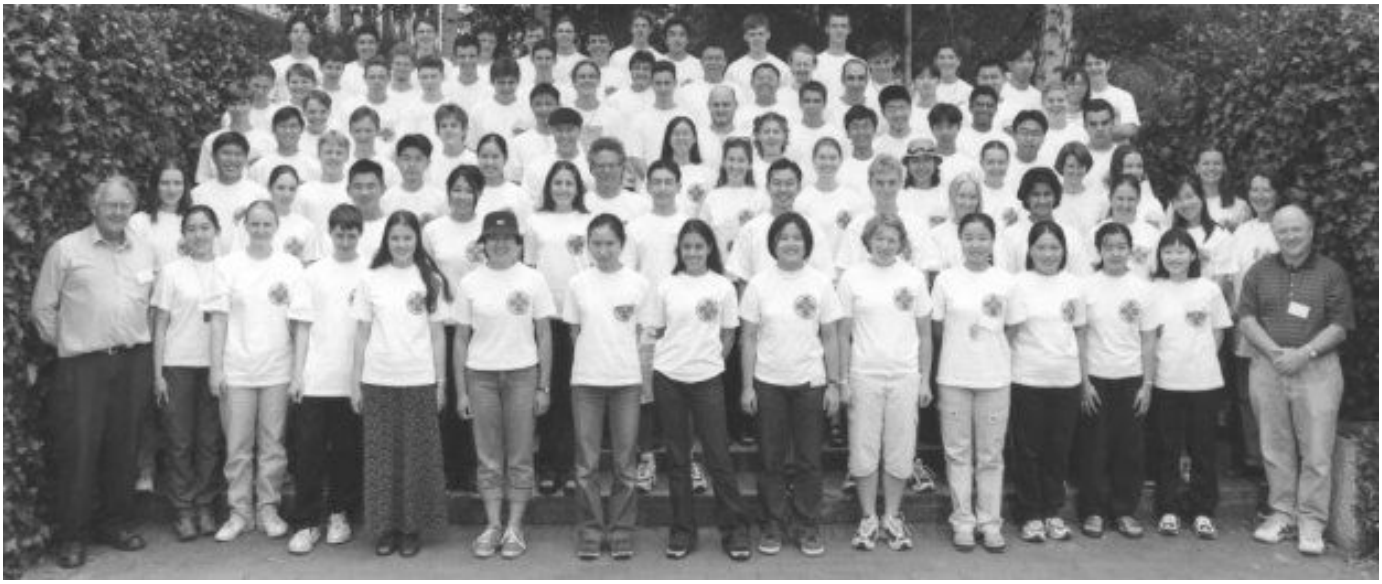
Arnold Ross

Terry Gagen

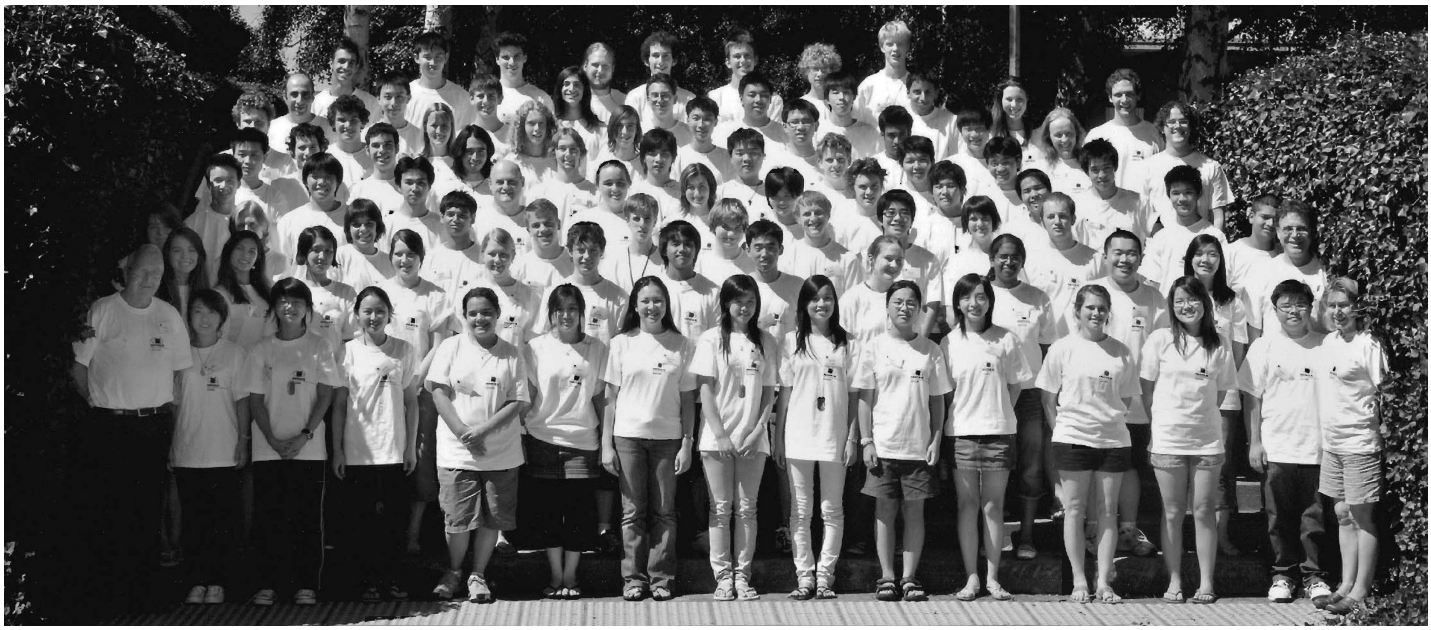
Bernhard Neumann

Hannah Neumann

Mike Newman



NMSS 2000



NMSS 2008

An Update from Mark Williams

I was privileged to attend NMSS in 1985. After that, I did an Electrical Engineering degree at Melbourne Uni. I started working as a professional Engineer in 1990. In 1995 I changed career slightly to work in an IT group, which I really enjoyed – it was a fascinating time. Now I work as a Business Analyst in marketing where I'm involved in data mining and analysis, system capability development and marketing methodologies. I find this such an interesting area, and am constantly learning. Right through my career, mathematics has been so important. Being able to problem solve, and apply mathematical solutions to real world problems has been so useful, and has certainly helped my career. I'm so thankful I was able to attend NMSS. Over 20 years later, I'm still impressed by the exposure to such diverse mathematical concepts I enjoyed at NMSS!

A Puzzle

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21 1 9 2 1 15 6 26 3 29 26 0 12 4 6 8 11 10 13 2 0 13 5 2 3 29 8 12
21 3 9 15 13 2 3 29 11 1 8 9 1 12 29 0 11 3 0 28 17 8 13 5 12 17 12
24 11 4 2 0 8 17 13 2 8 24 11 3 6 15 0 11 4 27 26 11 16 17 8 13 13 6
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19 27 29 4 24 11 17 3 19 25 11 16 11 9 11 7 5 2 6 9 8 12 20 27 2 10
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Do let me know if you crack it - Ed

Mathematics and Social Responsibility

I attended NMSS as a student in 1998 and 1999 and as a tutor in 2001 and 2006. The interests and relationships I developed there have stayed with me. I am pursuing a Ph.D. in pure mathematics at Sydney University. The maths department is filled with NMSS graduates and Terry's office is next to mine. Doing abstract mathematics is like putting together the pieces of the world's biggest jigsaw puzzle – it is mysterious, astounding and sometimes infuriating, and I love it! I am studying objects called unitary reflection groups, which describe the symmetries of objects in complex n-dimensional space. A better understanding of these groups is expected to have significant ramifications for Lie theory (continuous symmetry of mathematical structures), which infiltrates many areas of maths and theoretical physics.

The last two summers, however, I have been in South Africa. I run a charity called YOUTHCONNECTED which supports an HIV/AIDS organisation in the province of Kwa-Zulu Natal. I was there training carers and treating patients. This region has one of the highest infection rates in the world – up to 66% in some villages – and the country as a whole has around 6 million people living with HIV/AIDS.

In the midst of this and other humanitarian crises around the world I have often wondered how the type of work that I do, something so abstract and seemingly unconnected world affairs, relates to my broader social responsibility as a human being.

But is pure mathematics so unrelated to our everyday world? When Cardano discovered complex numbers in the 16th century, he said they were "as subtle as they are useless". Nowadays, physics and engineering, fields with clear social implications, would be lost without them! We can never know what use our discoveries will have decades and even centuries down the line, and the fact that time and again our abstract musings actually turn out to be useful is just part of the mystery.

We can't rely on this happening though, so pure mathematics must have some other value independent of its utility for its pursuit to be justified. Quite simply, it is beautiful, and gazing at the sheer wonder of the universe forms much of what gives my life meaning. If you don't see why mathematics is beautiful nobody can explain it to you – but then the same is true of a Picasso painting, or of a Beethoven symphony. Rather than burn our books (or our music, or our paintings), surely we can celebrate them and at the same time remain conscious of our fellow human beings whose struggle for survival precludes them from indulging in such luxuries? Indeed, one of the advantages of the academic life is the flexibility it brings me to indulge both passions at once.

When I work in so-called "disadvantaged" communities, I realise both how lucky I am and also how much our own society lacks. There is a real resilience, a joie de vivre in the face of immense difficulty which I see in the people I work with in Africa. Our education and our cleverness cannot buy that, and it is the mutual learning which makes these cross-cultural interactions so worthwhile.

Justin Koonin

CAN YOU HELP?

As you are aware, the NMSS needs all the support that you can give. The school is now without a major sponsor and only remains viable because of the donations of past students and their parents.

I urge you to make a tax deductible donation if at all possible. Cheques should be made out to the Australian National University (NMSS) and sent to Professor T M Gagen, Director NMSS, School of Mathematics and Statistics, University of Sydney, NSW 2006.

Thank you for your continued support of NMSS.